B.Sc V Sem physics Honors
End sem examination
paper : BP 502 (Optical Instruments & Techniques)
code : AS - 2779

Solutions :

Section - A

1. (i) (b) $\begin{bmatrix} 1 & 1/5 \\ 0 & 1 \end{bmatrix}$

(ii) (c) Both (a) and (b)

(iii) (b) $f_e < f_o$

(iv) (c) 3, 6, 9, ...

(v) (c) Both (a) and (b)

(vi) (b) 0.0254 cm

(vii) (c) order of the spectrum

(viii) (b) increases

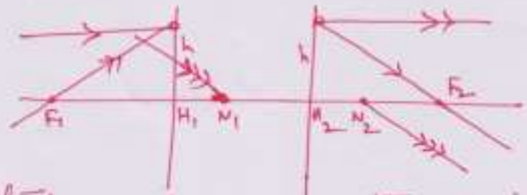
(ix) (c) 90°

(x) (b) e-ray

Section - B

2. There are six cardinal points in a lens system. They are :

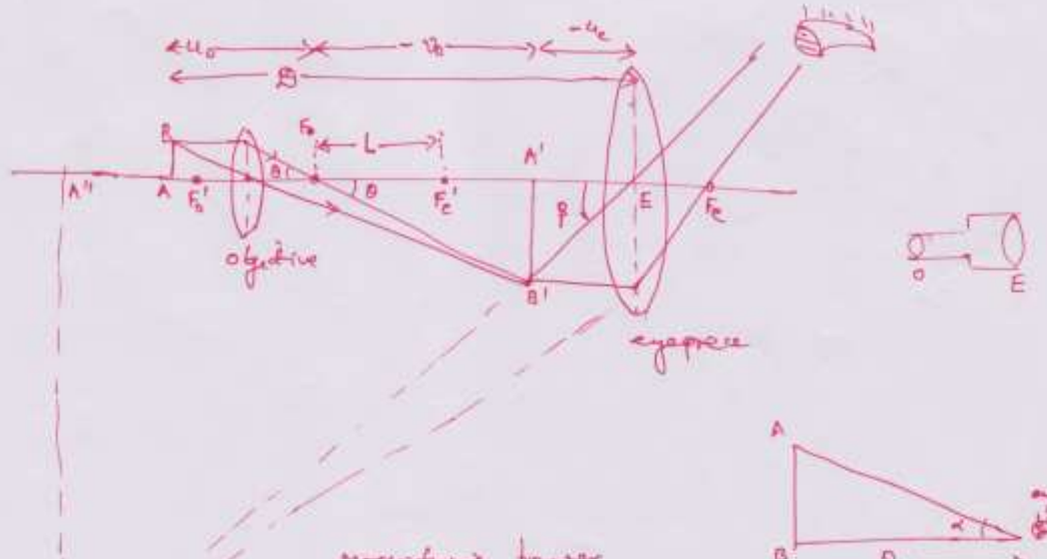
Two principal or unit points : The principal points H_1 & H_2 are a pair of conjugate points, on the principal axis for which linear transverse magnification is unity and positive. If an object is placed at one principal point, then an image of the same size is formed at the other principal point. The planes passing through these points & perpendicular to the principal axis are called the principal planes. So, when a ray strikes the first unit plane at a certain height, it emerges out from the second unit plane at the same height on the other side.



<u>Two focal points</u>: parallel rays after passing through the lens system meet at a point on the axis. There are two such points F_1 & F_2 on either side of the system. The first focal point F_1 is an object point on the principal axis for which the image point lies at infinity. Similarly, the second focal point F_2 is an image point on the principal axis for which the object lies at infinity. The planes passing through these points are known as focal planes.

<u>Two nodal points</u>: These are a pair of conjugate points having unit positive angular magnification. They are denoted by N_1 & N_2. They lie on the principal axis. If a ray is directed towards the first nodal point N_1, the ray emerges from N_2 parallel to the original direction. planes passing through the nodal points & perpendicular to the principal axis are known as nodal planes. If the medium is same on the both sides, then the respective nodal points coincide with the principal points.

3.



Magnifying power

$$M = \frac{\beta}{\alpha}$$

$$\alpha = \tan\alpha = \frac{AB}{EA} \qquad EA = -D$$

$$\Rightarrow \quad \alpha = AB/-D$$

from figure, $\beta = \tan\beta = \frac{A'B'}{EA'}$ $\quad EA' = -u_e$

$$\Rightarrow \quad \beta = \frac{A'B'}{(-u_e)}$$

$$\therefore \quad M = \frac{\beta}{\alpha} = \frac{A'B'}{(-u_e)} \bigg/ \left(\frac{AB}{-D}\right)$$

$$M = \frac{A'B'}{AB} \cdot \frac{D}{u_e} \qquad \text{But } \frac{A'B'}{AB} = \left(\frac{v_o}{-u_o}\right)$$

$$M = -\frac{v_o}{u_o} \cdot \frac{D}{u_e} \longrightarrow (1)$$

By using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ Here $f = f_e$, $v = -v_e$, $u = -u_e$

$$\Rightarrow \quad \frac{1}{f_e} = \frac{1}{-v_e} - \frac{1}{(-u_e)}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{v_e}$$

$$\therefore (1) \Rightarrow \quad M = \frac{-v_o}{u_o} \cdot D \cdot \left[\frac{1}{f_e} + \frac{1}{v_e}\right]$$

$$\boxed{M = \frac{-v_o}{u_o}\left[\frac{D}{f_e} + \frac{D}{v_e}\right]}$$

case i: If $v_e = \infty \Rightarrow M = \frac{-v_o}{u_o}\frac{D}{f_e}$

case ii: If $v_e = D \Rightarrow M = \frac{-v_o}{u_o}\left[1 + \frac{D}{f_e}\right]$

$\therefore$ M can be increased if f_o is small

f_e is small

tube length L should be large.

4. Overlapping spectra: The n^{th} principal maxima for wavelength λ is

$$(a+b)\sin\theta = n\lambda$$

Thus when the incident light has a large no. of wavelengths $\lambda_1, \lambda_2, \cdots \lambda_n$ in decreasing order, the spectral lines will have the same angle of diffraction i.e they will overlap if

$$(a+b)\sin\theta = 1 \cdot \lambda_1 = 2 \cdot \lambda_2 = 3 \cdot \lambda_3 = \cdots = n \cdot \lambda_n.$$

ex: The red in 3rd order, the green in 4th order & the violet in 5th order coincide because

$(a+b)\sin\theta = 3\times 7000\times 10^{-10} = 4\times 5250\times 10^{-10} = 5\times 4200\times 10^{-10}$

Here $(a+b)$ in metres.

Absent spectra:

directions of principal maxima $\Rightarrow (a+b)\sin\theta = n\lambda$

directions of minima due to single slit $\Rightarrow a\sin\theta = m\lambda \quad m = 1,2,3,\ldots$

If the above two equations are simultaneously satisfied, then the principal maxima of order n will not be present in the grating spectrum.

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n\lambda}{m\lambda}$$

$$\Rightarrow \boxed{n = \left(\frac{a+b}{a}\right) m}$$

If $b=a \Rightarrow n = 2m = 2,4,6,\ldots$ order spectra will be absent.

If $b = 2a \Rightarrow n = 3m = 3,6,9,\ldots$ order spectra will be absent.

Now we know, $(a+b)\sin\theta_n = n\lambda$

$$\Rightarrow n = \frac{(a+b)\sin\theta_n}{\lambda}$$

$$n_{max} = \frac{a+b}{\lambda} \quad \because \theta = 90^\circ$$

When the angle of incidence is $i \Rightarrow$

$$n = \frac{(a+b)(\sin\theta_m + \sin i)}{\lambda}$$

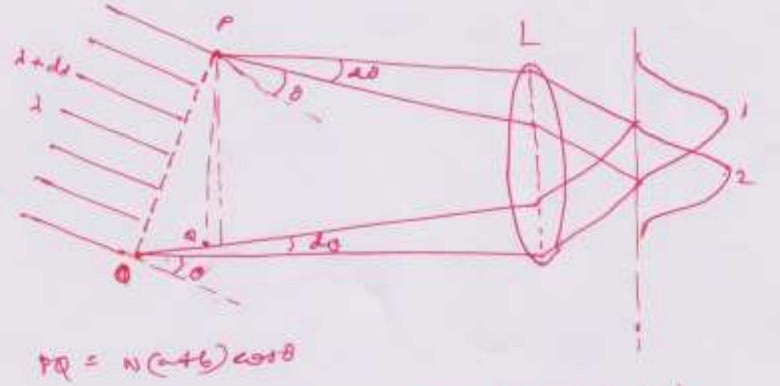
$$\Rightarrow n_{max} = \frac{(a+b)(1+\sin i)}{\lambda}$$

When the grating element $(a+b) < 2\lambda$, for normal incidence we have

$$n_{max} < \frac{2\lambda}{\lambda} < 2$$

This means that for normal incidence when $(a+b) < 2\lambda$, only first order is obtained

5. The ability of an optical instrument to just resolve the images of two nearby point sources is called its resolving power.



$PQ = N(a+b)\cos\theta$

n^{th} principal maxima $\Rightarrow (a+b)\sin\theta = n\lambda$

The first minima adjacent to the n^{th} maxima

$\Rightarrow \quad N(a+b)\sin(\theta+d\theta) = m\lambda$ where m is an integer except $0, N, 2N, \cdots nN$.

$$\therefore \quad N(a+b)\sin(\theta+d\theta) = (Nn+1)\lambda \longrightarrow (1)$$

According to Rayleigh's criterion, the two spectral lines of wave-lengths λ & $\lambda+d\lambda$ are just resolved when n^{th} maxima of wavelength $\lambda+d\lambda$ falls on first minima of wavelength λ adjacent to its n^{th} maxima.

$\therefore$ for n^{th} maxima of wavelength $(\lambda+d\lambda)$, we have

$$(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)$$

or $$N(a+b)\sin(\theta+d\theta) = Nn(\lambda+d\lambda) \longrightarrow (2)$$

$\therefore$ from (1) & (2) $\Rightarrow \quad (Nn+1)\lambda = Nn(\lambda+d\lambda)$

$\Rightarrow$ RP of grating $\boxed{\frac{\lambda}{d\lambda} = nN}$

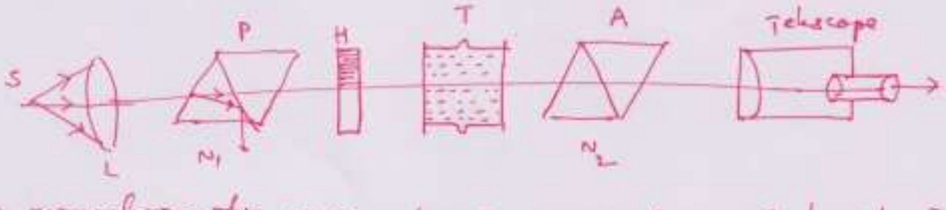
$$\Rightarrow \frac{\lambda}{d\lambda} = N\times n\,(a+b)\cos\theta \times \frac{1}{(a+b)\cos\theta}$$

$$= \underbrace{N(a+b)\cos\theta}_{\text{aperture}} \times \underbrace{\frac{n}{(a+b)\cos\theta}}_{\text{dispersive power}}$$

$\Rightarrow$ RP of grating $= \frac{\lambda}{d\lambda} = \lambda \times \frac{d\theta}{d\lambda}$

$$\boxed{RP = nN = \frac{N(a+b)\sin\theta}{\lambda}}$$

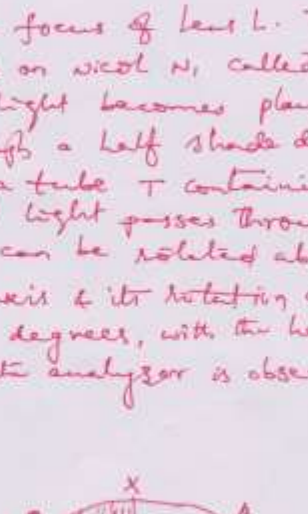
6. Apparatus :



S is a monochromatic source placed at the focus of lens L. The beam rendered parallel by lens L, falls on nicol N_1 called polarizer. After passing through P, the light becomes plane polarized. This polarised light passes through a half shade device H called Laurent's plate & then through a tube T containing optically active solution. The transmitted light passes through another nicol N_2 called analyzer A which can be rotated about the direction of propagation of light as axis & its rotation can be read on a circular scale graduated in degrees, with the help of a vernier. The light emerging from the analyzer is observed through a telescope.

Action of Laurent's half shade plate :

Half shade plate H is combination of two semicircular plates XBY & XDY. The plate XDY is of Quartz & is cut parallel to optic axis while the plate XBY is of glass. Both the plates joined along the diameter XY. The thickness of the quartz plate is such that it introduces a path difference $\lambda/2$ or a phase difference π between o-ray & e-ray vibrations i.e., it is a half wave plate. The thickness of the glass plate is such that it absorbs the same amount of light as the quartz plate.



Let CP be the direction of vibrations in the plane polarised light. The light passing through glass remains unaffected. But the light falling on Quartz is broken into two components parallel & lar to optic axis XCY.

O-component is $\perp$ar to optic axis i.e., along CB while the e-component is parallel to optic axis i.e., along CX. As in quartz o-component travels faster, hence passing through the Quartz it gains a path of $\lambda/2$ or phase of π over e-component. Hence on emergence from the Quartz o-component has vibrations along CB. The e-component has vibrations still along CX. Thus the light emerging from the quartz plate has resultant vibration along CQ where $\angle PCX = \angle QCX = \theta$.

$\Rightarrow \quad \angle PCQ = 2\theta$.

principal plane of analyser is parallel to QCQ' $\Rightarrow$ left half will be brighter

principal plane of analyser is parallel to PCP' $\Rightarrow$ right half will be brighter

principal plane of analyser is parallel to XCY $\Rightarrow$ two halves of the field appear equally illuminated.

Determination of specific rotation of sugar solution:

The tube is first filled with water & analyser is adjusted to obtain equally illuminated position of field of view. Now the sugar solution of known concentration is filled in the tube T. The solution rotates the plane of vibration CP & CQ through the same angle in the same direction. Sugar solution rotates the plane of vibration in clockwise direction. Now the analyser is rotated in the clockwise direction to obtain equally illuminated position of the field of view again. The difference between the two analyser readings gives the angle of rotation θ. In the experiment, solutions of various known concentrations are taken & corresponding angles of rotation are measured. Then a graph is plotted between concentration c & and the angle of rotation θ. The graph is a straight line.

$\Rightarrow$ specific rotation

$$\alpha_\lambda^t = \frac{10\,\theta}{l c}$$

$$\Rightarrow \alpha = \frac{10}{l}(\text{slope})$$

$l \rightarrow$ length of the tube in cm.
$\theta \rightarrow$ degree
$c \rightarrow$ gm/cc.

7. $$S = \begin{bmatrix} 1 & -P_1 - P_2 \\ 0 & 1 \end{bmatrix}$$

Here $$P_1 = P_2 = \frac{\mu - 1}{R_1} = \frac{\mu - 1}{|R_2|} = \frac{1.5 - 1}{50} = \frac{0.5}{50} = 0.01$$

$$\therefore \quad S = \begin{bmatrix} 1 & -0.01 - 0.01 \\ 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & -0.02 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad f = \frac{1}{0.02}$$

$$f = 50\,cm$$

8. The angular positions of minima are $a \sin\theta = m\lambda$

$$\Rightarrow \sin\theta = \frac{m\lambda}{a}$$

$a = 2.2 \times 10^{-6}\,m$ & $\lambda = 5500 \times 10^{-10}\,m$

for first order: $m = 1$

$$\Rightarrow \sin\theta_1 = \frac{1\lambda}{a} = \frac{5500 \times 10^{-10}}{2.2 \times 10^{-6}} = 0.25$$

$$\Rightarrow \theta_1 = \sin^{-1}(0.25) = 14.47^\circ$$

for second order: $m = 2$

$$\Rightarrow \sin\theta_2 = \frac{2\lambda}{a} = \frac{2 \times 5500 \times 10^{-10}}{2.2 \times 10^{-6}} = 0.5$$

$$\Rightarrow \theta_2 = \sin^{-1}(0.5) = 30^\circ$$